Worksheet 3

Recall: A **rational number ($\mathbb{Q}$)** is a number that can be expressed as the quotient or fraction $\frac{p}{q}$ of two integers, a numerator p and a non-zero denominator q. The **irrational numbers ($\mathbb{I}$)** are all the real numbers which are not rational.

1. Prove that the sum of a rational number and an irrational number is irrational
    a. What can we suppose:

    b. Write the statement in mathematical symbols

    c. Write down an example to support the statement

    d. What proof method are we considering using:
        i. Direct proof? (if not, why?)

        ii. Proof by contrapositive? (if not, why?)

        iii. Proof by contradiction? (if not, why?)

    e. Proof:

2. Prove that if a and b are consecutive integers, then the sum $a + b$ is odd
   a. What can we suppose:
   b. Write the statement in mathematical symbols
   c. Write down an example to support the statement
   d. What proof method are we considering using:
      i. Direct proof? (if not, why?)
      ii. Proof by contrapositive? (if not, why?)
      iii. Proof by contradiction? (if not, why?)
   e. Proof:

3. Prove that the real number $\sqrt{4}$ is rational
   a. What can we suppose:
   b. Definitions we could use:
   c. What proof method are we considering using:
      i. Direct proof? (if not, why?)
      ii. Proof by contrapositive? (if not, why?)
      iii. Proof by contradiction? (if not, why?)
   d. Proof:

4. Prove that the real number $\sqrt{2}$ is irrational

   a. What can we suppose:

   b. What proof method are we considering using:

      i. Direct proof? (if not, why?)

      ii. Proof by contrapositive? (if not, why?)

      iii. Proof by contradiction? (if not, why?)

   c. Proof:

5. Suppose $n \in \mathbb{Z}$ (integers), prove that $(n+1)^2 - 1$ is even if and only if n is even

6. Let $a, b, c \in \mathbb{Z}$, prove that if a and c are odd, then $ab + bc$ is even

7. Let $x, y \in \mathbb{Z}$, prove that if $3 \nmid x$ and $3 \nmid y$, then $3 | (x^2 + y^2)$